31. $\mathbb{E}_{X_n}(X_n) = \mathbb{E}_{w_1,\dots,w_n}(w_1+\cdots+w_n)$

$= \mathbb{E}_{w_1}(w_1) + \cdots + \mathbb{E}_{w_n}(w_n)$ つねに成立.

$= n\cdot\mathbb{E}_W(W) = n(p-q)$

$\mathbb{V}_{X_n}(X_n) = \mathbb{V}_{w_1,\dots,w_n}(w_1+\cdots+w_n)$

$= \mathbb{V}_{w_1}(w_1) + \cdots + \mathbb{V}_{w_n}(w_n)$ $w_1,\dots,w_n$ は独立

$= n\,\mathbb{V}_W(W) = 4npq$

32. $n < m$ とする. このとき,

つねに成立.

$$\mathbb{E}_{X_n,X_m}(X_nX_m) = \mathbb{E}_{w_1,\dots,w_m}\left[\sum_{i=1}^{n} w_i \sum_{j=1}^{m} w_j\right] = \sum_{i=1}^{n}\sum_{j=1}^{m}\mathbb{E}_{w_i,w_j}[w_iw_j]$$

$$= \sum_{i=1}^{n}\mathbb{E}_{w_i}[w_i^2] + \sum_{i\neq j}\mathbb{E}_{w_i,w_j}(w_iw_j) = n + \sum_{i\neq j}\mathbb{E}_{w_i}(w_i)\cdot\mathbb{E}_{w_j}(w_j)$$

$$= n + n(m-1)\cdot(p-q)^2$$

$\therefore\ \mathrm{Cov}(X_n, X_m) = n + n(m-1)\cdot(p-q)^2 - n(p-q)\cdot m(p-q) = 4npq.$

33. n 回中, m 回, $+1$ が出るとすると, $X_n = (+1)m + (-1)(n-m) = 2m-n.$

ゆえに, $X_n = k$ となる m は

$$2m - n = k \qquad \therefore\ m = \frac{n+k}{2}$$

$$\therefore\ \mathbb{P}(\{X_n = k\}) = \mathbb{P}\left(\left\{n\text{ 回中}, \tfrac{n+k}{2}\text{ 回}, +1\text{ が出る}\right\}\right)$$

$$= {}_nC_{\frac{n+k}{2}}\, p^{\frac{n+k}{2}} (1-p)^{n-\frac{n+k}{2}} = {}_nC_{\frac{n+k}{2}}\, p^{\frac{n+k}{2}} q^{\frac{n-k}{2}}$$

ただしこれは $\frac{n+k}{2}$ が整数となるときで, そうでないときは $\mathbb{P}(\{X_n = k\}) = 0.$

$n = 3$ のとき,

$$\begin{cases} \mathbb{P}(\{X_3 = 3\}) = {}_3C_3\, p^3 q^0 = p^3 \\ \mathbb{P}(\{X_3 = 1\}) = {}_3C_2\, p^2 q^1 = 3p^2q \\ \mathbb{P}(\{X_3 = -1\}) = {}_3C_1\, p q^2 = 3pq^2 \\ \mathbb{P}(\{X_3 = -3\}) = {}_3C_0\, p^0 q^3 = q^3 \end{cases}$$

これ以外の k に対しては $\mathbb{P}(\{X_3 = k\}) = 0.$

34. $\pi_n(M) = \mathbb{P}(\{X_n = M\})$

$= \mathbb{P}(\{X_n = M\} \cap \{W_n = 1\})$

$+ \mathbb{P}(\{X_n = M\} \cap \{W_n = -1\})$

$= \mathbb{P}(\{X_{n-1} = M-1\} \cap \{W_n = 1\}) + \mathbb{P}(\{X_{n-1} = M\})$

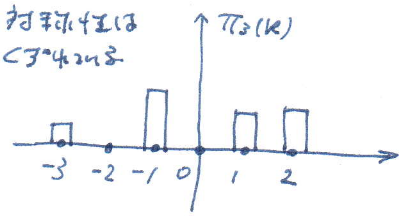
$= \pi_{n-1}(M-1) \cdot p + \pi_{n-1}(M)$

$\pi_n(M-1) = \mathbb{P}(\{X_n = M-1\} \cap \{W_n = 1\}) + \mathbb{P}(\{X_n = M-1\} \cap \{W_n = -1\})$

$= \mathbb{P}(\{X_{n-1} = M-2\} \cap \{W_n = 1\}) + 0$

$= p \cdot \pi_{n-1}(M-2)$

対称性はくずれる



ゆえに, $\pi_n(2) = p\,\pi_{n-1}(1) + \pi_{n-1}(2)$, $\pi_n(1) = p\,\pi_{n-1}(0)$ が成立.

$\pi_3(2) = p\,\pi_2(1) + \pi_2(2)$

$= p \cdot p\,\pi_1(0) + [\,p\,\pi_1(1) + \pi_1(2)\,]$

$= p^2 [\,\underbrace{\pi_0(-1)}_{=0} + q\,\underbrace{\pi_0(1)}_{=0}\,] + p \cdot p\,\underbrace{\pi_0(0)}_{=1} + [\,p\,\underbrace{\pi_0(1)}_{=0} + \underbrace{\pi_0(2)}_{=0}\,] = p^2.$

同様に, $\pi_3(1) = 2p^2q$, $\pi_3(0) = 0$, $\pi_3(-1) = 3pq^2$, $\pi_3(-2) = 0$, $\pi_3(-3) = q^3$

35. $\pi_n(1) = \mathbb{P}(\{X_n = 1\}) = \mathbb{P}(\{X_n = 1\} \cap \{W_n = 1\}) + \mathbb{P}(\{X_n = 1\} \cap \{W_n = -1\})$

$= \mathbb{P}(\{X_{n-1} = 0\}) + \mathbb{P}(\{X_{n-1} = 2\} \cap \{W_n = -1\})$

$= \mathbb{P}(\{X_{n-1} = 0\}) + \mathbb{P}(\{X_{n-1} = 2\}) \cdot \mathbb{P}(\{W_n = -1\})$

$= \pi_{n-1}(0) + q \cdot \pi_{n-1}(2)$

$\pi_n(0) = \mathbb{P}(\{X_n = 0\}) = \mathbb{P}(\{X_{n-1} = 1\} \cap \{W_n = -1\}) = \mathbb{P}(\{X_{n-1} = 1\}) \cdot \mathbb{P}(\{W_n = -1\})$

$= q \cdot \pi_{n-1}(1)$

$k \geq 2$ に対しては今までと同様, $\pi_n(k) = p \cdot \pi_{n-1}(k-1) + q \cdot \pi_{n-1}(k+1)$.

$\pi_3(1) = \pi_2(0) + q\,\pi_2(2) = q \cdot \pi_1(1) + q \cdot [\,p \cdot \pi_1(1) + q \cdot \pi_1(3)\,]$

$= (q + pq)[\,\underbrace{\pi_0(0)}_{=0} + q \cdot \underbrace{\pi_0(2)}_{=0}\,] + q^2 [\,p\,\underbrace{\pi_0(2)}_{=0} + q \cdot \underbrace{\pi_0(4)}_{=0}\,] = 0.$