応用確率論 - 問題例解答

1. (1) 1回目に赤玉を取り出し，2回目に白玉を取り出す，という根元事象を

$\omega = R \to W$ などと表すと，

$$\Omega = \{ R\to R,\ R\to W,\ W\to R,\ W\to W \}$$

(2)

$\mathcal{F}_1'$	$\{R\to R\}$	$\{R\to W, W\to R\}$	$\{W\to W\}$
$\mathbb{P}$	$\frac{7}{10}\cdot\frac{7}{10}=\frac{49}{100}$	$\frac{7}{10}\cdot\frac{3}{10}+\frac{3}{10}\cdot\frac{7}{10}=\frac{42}{100}$	$\frac{3}{10}\cdot\frac{3}{10}=\frac{9}{100}$

(3)

$\mathcal{F}_2'$	$\{R\to R\}$	$\{R\to W, W\to R, W\to W\}$
$\mathbb{P}$	$\frac{7}{10}\cdot\frac{7}{10}=\frac{49}{100}$	$1-\frac{49}{100}=\frac{51}{100}$

2. (1) 問1.の(1)と同じ

(2)

$\mathcal{F}_1'$	$\{R\to R\}$	$\{R\to W, W\to R\}$	$\{W\to W\}$
$\mathbb{P}$	$\frac{7}{10}\cdot\frac{6}{9}=\frac{42}{90}$	$\frac{7}{10}\cdot\frac{3}{9}+\frac{3}{10}\cdot\frac{7}{9}=\frac{42}{90}$	$\frac{3}{10}\cdot\frac{2}{9}=\frac{6}{90}$

(3)

$\mathcal{F}_2'$	$\{R\to R\}$	$\{R\to W, W\to R, W\to W\}$
$\mathbb{P}$	$\frac{7}{10}\cdot\frac{6}{9}=\frac{42}{90}\fallingdotseq 0.47$	$1-\frac{42}{90}=\frac{48}{90}\fallingdotseq 0.53$

3. (1) $p_5 = {}_{500}C_5 \times (0.01)^5 (0.99)^{495} \fallingdotseq 0.1764$

(2)
$$P_o(k,\lambda) = \lim_{N\to\infty} \frac{N!}{(N-k)!\,k!}\left(\frac{\lambda}{N}\right)^k \left(1-\frac{\lambda}{N}\right)^{-k}\left(1-\frac{\lambda}{N}\right)^N$$

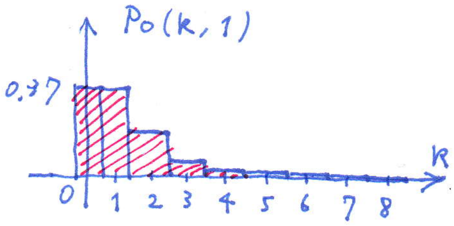
$$= \lim_{N\to\infty} \frac{N\cdot(N-1)\cdots\cdots(N-k+1)}{k!}\left(\frac{\frac{\lambda}{N}}{1-\frac{\lambda}{N}}\right)^k \left(1-\frac{\lambda}{N}\right)^N$$

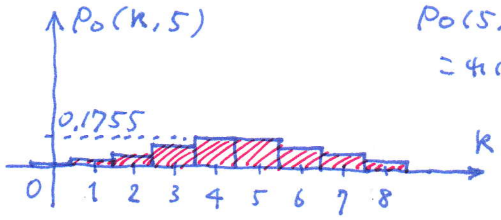
$$= \lim_{N\to\infty} \frac{N\cdot(N-1)\cdot\cdots\cdot(N-k+1)}{(N-\lambda)\cdot(N-\lambda)\cdot\cdots\cdots(N-\lambda)}\cdot\frac{\lambda^k}{k!}\cdot\left(1-\frac{\lambda}{N}\right)^N$$

$$= \lim_{N\to\infty}\left(\frac{1}{1-\frac{\lambda}{N}}\right)\left(\frac{1-\frac{1}{N}}{1-\frac{\lambda}{N}}\right)\cdots\left(\frac{1-\frac{k-1}{N}}{1-\frac{\lambda}{N}}\right)\cdot\frac{\lambda^k}{k!}\cdot\left\{\left(1-\frac{\lambda}{N}\right)^{-\frac{N}{\lambda}}\right\}^{-\lambda}$$

$$= \frac{\lambda^k}{k!}e^{-\lambda}$$

(3)



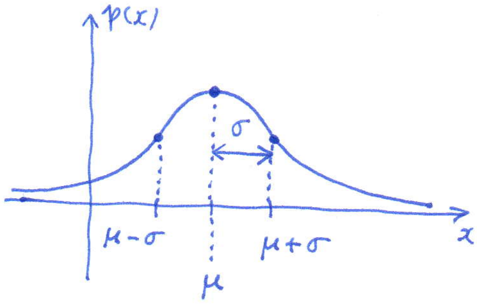


$Po(5,5) \fallingdotseq 0.1755$ で
これは $p_5 \fallingdotseq 0.1764$ と
非常に近い。

4. (1) $p'(x) = \dfrac{\mu - x}{\sqrt{2\pi}\,\sigma^3} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ ， $p''(x) = \dfrac{e^{-\frac{1}{2\sigma^2}(x-\mu)^2}}{\sqrt{2\pi}\,\sigma^5} \cdot \left\{ (x-\mu)^2 - \sigma^2 \right\}$

より，極大点は $x = \mu$（最大値をとる），変曲点は $x = \mu \pm \sigma$

(2) $\dfrac{1}{\sqrt{2}\sigma}(x-\mu) = y$ とおくと，



$$\int_{-\infty}^{\infty} p(x)\,dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-y^2} \cdot \sqrt{2}\sigma\,dy$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-y^2}\,dy = \frac{\sqrt{\pi}}{\sqrt{\pi}} = 1$$

5. (1) $\displaystyle\int_{-\infty}^{\infty} p(x)\,dx = c\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = 1$ が規格化条件． $x = \tan\theta$ とおくと， $\dfrac{dx}{d\theta} = \dfrac{1}{\cos^2\theta}$ で，

θ は $-\dfrac{\pi}{2}$ から $\dfrac{\pi}{2}$ まで動く．ゆえに $\displaystyle c\int_{-\pi/2}^{\pi/2} \frac{1}{1+\tan^2\theta} \cdot \frac{d\theta}{\cos^2\theta} = c\int_{-\pi/2}^{\pi/2} d\theta = c\pi = 1 \quad \therefore c = \frac{1}{\pi}$

(2) $\displaystyle \mathbb{P}([0,1]) = \int_0^1 p(x)\,dx = \frac{1}{\pi}\int_0^1 \frac{dx}{1+x^2} = \frac{1}{\pi}\int_0^{\pi/4} \frac{1}{1+\tan^2\theta} \cdot \frac{d\theta}{\cos^2\theta} = \frac{1}{4}$